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# AXISYMMETRIC ELECTROVACUUM SPACETIMES WITH AN ADDITIONAL KILLING VECTOR AND RADIATION

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In the present note we briefly summarize our recent work[1,2] on possible additional symmetries of axially symmetric electrovacuum spacetimes which admit radiation. The main result states that only boost and rotation (axially) symmetric electrovacuum spacetimes can be radiative and asymptotically flat at null infinity $\mathcal{J}$ which admits global sections. If an additional symmetry is a translational spacelike or null Killing field the spacetime represents cylindrical or plane-type waves, local $\mathcal{J}$ may still exist but some of its generators are missing. Boost-rotation symmetric spacetimes are the only known exact explicit radiative solutions of Einstein's equations describing moving objects – singularities or black holes uniformly accelerated along the axis of symmetry. They are radiative and admit a smooth $\mathcal{J}$ although at least four points of $\mathcal{J}$ are missing. They represent the only known examples in which *arbitrarily strong* initial data with the given symmetry can be chosen on a hyperboloidal hypersurface which evolve into a complete, smooth null infinity and regular timelike infinity. For the latest reviews, containing a number of relevant references, see [3, 4].

Most recently the analysis of asymptotic symmetries was extended to spacetimes with null dust[5] and to spacetimes with polyhomogeneous $\mathcal{J}$[6].

In Refs. [1, 2] we assume that in addition to bound systems and radiation, the axisymmetric spacetimes may contain an infinite (cosmic) string along the axis of symmetry. Here we assume spacetimes without strings. Our main results can then be summarized in the following three theorems, the full versions of which and detailed proofs are given in Refs. [1] and [2].

**Theorem 1** *Suppose that an axially symmetric electrovacuum spacetime admits a "piece" of $\mathcal{J}^+$ in the sense that it admits the Bondi-Sachs expansions of the metric and electromagnetic field in coordinates $\{u, r, \theta, \phi\}$. Assume the spacetime admits an additional Killing vector $\eta$ forming with the axial Killing vector a two-dimensional Lie algebra (Killing vectors need not be hypersurface orthogonal) and the electromagnetic field has the same symmetry. Then the additional Killing vector has asymptotically the form*

$$\eta^\alpha = [-ku\cos\theta + \alpha(\theta),\ kr\cos\theta + \mathcal{O}(r^0),\ -k\sin\theta + \mathcal{O}(r^{-1}),\ \mathcal{O}(r^{-1})]\ , \qquad (1)$$

*where $k$ is a constant and $\alpha$ an arbitrary function of $\theta$. For $k = 0$ it generates asymptotically translations. For $k \neq 0$ it is the boost Killing field (one can put, without loss of generality, $\alpha = 0$ and $k = 1$) which generates the Lorentz*

*transformations along the axis of axial symmetry.*

**1) Translational Killing vectors:**

**Theorem 2** *If an axisymmetric electrovacuum spacetime admits a local* $\mathcal{J}^+$ *and an asymptotically spacelike translational Killing vector, and if the spacetime is not flat in the neighbourhood of* $\mathcal{J}^+$ *(i.e. the Bondi mass aspect* $M$ *or other metric functions are non-vanishing) then* $\mathcal{J}^+$ *has singular generators at* $\theta = \theta_0 \neq 0,\ \pi$ *and the spacetime contains cylindrical waves. If the translational Killing vector is null, then* $\mathcal{J}^+$ *is singular at* $\theta = 0$ *or* $\pi$ *– a wave propagating along the symmetry axis is present.*

**Theorem 3** *If an axisymmetric electrovacuum spacetime with a non-vanishing Bondi mass* $m$ *admits an asymptotically translational Killing vector and a complete cross section of* $\mathcal{J}^+$*, then the translational Killing vector is timelike and spacetime is thus stationary.*

**2) The boost Killing vector:**

We now find non-vanishing news functions, $c_{,u}$, $d_{,u}$ (describing gravitational radiation) and $X$, $Y$ (describing electromagnetic radiation) to have the forms:

$$c_{,u}(u,\theta)=\frac{K(w)}{u^2},\ d_{,u}(u,\theta)=\frac{L(w)}{u^2},\ X(u,\theta)=\frac{\mathcal{E}(w)}{u^2},\ Y(u,\theta)=\frac{\tilde{\mathcal{B}}(w)}{u^2}, \tag{2}$$

where $K$ etc are functions of $w = \sin\theta/u$. They determine the mass aspect $M$, other metric and field functions and the Bondi mass:

$$M(u,\theta)=\frac{(w^2K)_{,w}}{2\sin\theta}+\frac{\lambda(w)}{w^3u^3} \tag{3}$$

where

$$\lambda_{,w}=w^2(K^2+L^2+\mathcal{E}^2+\tilde{\mathcal{B}}^2)-\frac{(w^3K_{,w})_{,w}}{2w}\ . \tag{4}$$

The Weyl tensor has, in general, radiative components ($\sim 1/r$). The boost-rotation symmetric spacetimes are clearly radiative[3,4].

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